As stated in the May Number, the published solution conforms to the solution of Problem IX, page 52, of Meyer's *Wahrscheinlichkeitsrechnung*, which problem I take to be similar to the one under consideration. The formula used by Professor Landis in his solution is the same as that obtained by Meyer, a formula which is only approximate, but holds with great exactness if n is large. The correct result is to be obtained from the equation

$$\tfrac{1}{2}=1-n\left(\frac{n-1}{n}\right)^i+\frac{n(n-1)}{2!}\left(\frac{n-2}{n}\right)^i-\frac{n(n-1)(n-2)}{3!}\left(\frac{n-3}{n}\right)^i$$
$$+\frac{n(n-1)(n-2)(n-3)}{4!}\left(\frac{n-4}{4}\right)^i+,\text{ etc.},$$

where n is the number of numbers and i the number of drawings. This is Meyer's answer to his Problem VIII, which reads: Eine Lotterie besteht aus n Nummern, in jeder Ziehung wird eine davon gezogen. Es soll die Wahrscheinlichkeit π gefunden werden, dass in i Ziehungen alle nummern erschienen sind.

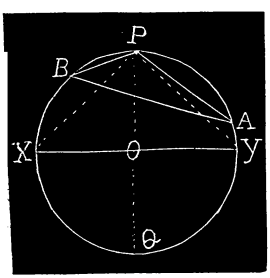
Professor Matz insists that the solution of Problem 158 in the May Number of the MONTHLY is also incorrect, his contention being that to take an arc for every point on AC and BD would not be taking them uniformly on AB. In reference to this contention, we must again call attention to Dr. Moore's *Note on Mean Value*, page 303, Vol. II of MONTHLY. The problem as stated is indefinite, and thus one may choose any law of distribution he wishes. Accordingly, the published solution is correct according to the law of distribution chosen. Other laws of distribution may be chosen, giving other results. ED. F.

161. Proposed by F. P. MATZ. Sc. D., Ph. D., Reading, Pa.

A triangle is inscribed at random in a circle; (a) what is the chance the triangle is *oblique;* and (b) what is the chance the triangle is *less in area* than $\frac{1}{4}\pi r^2$?

Solution by the PROPOSER.

(I) Put $OP=r$, $\angle APO=\theta$, $\angle BPO=\phi$; then $\triangle APB=2r^2\cos\theta\cos\phi \times\sin(\theta+\phi)$. In order that $\triangle APB$ may be *obtuse*-angled at P, the point A is restricted to the arc PAX, and the point B to the arc PBY. The required chance, therefore, becomes



$$C_1=\frac{2r^2\int_{\frac{1}{4}\pi}^{\frac{1}{2}\pi}\int_{\frac{1}{4}\pi}^{\phi}\cos\theta\cos\phi\sin(\theta+\phi)d\phi d\theta}{2r^2\int_{0}^{\frac{1}{2}\pi}\int_{0}^{\phi}\cos\theta\cos\phi\sin(\theta+\phi)d\phi d\theta}$$

$$=\frac{\int_{\frac{1}{4}\pi}^{\frac{1}{2}\pi}[2\sin^2\phi\cos^2\phi-\frac{1}{2}\cos^2\phi+(\phi-\frac{1}{4}\pi-\frac{1}{2})\sin\phi\cos\phi]d\phi}{\int_{0}^{\frac{1}{2}\pi}[2\sin^2\phi\cos^2\phi+\phi\sin\phi\cos\phi]d\phi}$$

$$=\left(\frac{\pi}{16}-\tfrac{1}{8}\right)/\frac{\pi}{4}=\tfrac{1}{2}\left(\tfrac{1}{2}-\frac{1}{\pi}\right).$$

(II) In this case the *superior* limit of ϕ in the numerator of C_1 is the value of ϕ derived from the equation $\sin\phi\cos^3\phi=\frac{1}{16}\pi$; and the *inferior* limit of the same variable is zero.

The required chance C_2 can, therefore, be found approximately; but is not of sufficient interest to warrant the labor required to find it.

MISCELLANEOUS.

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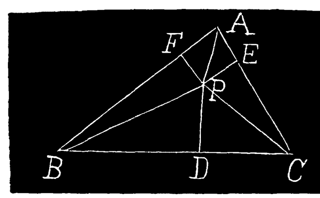
147. Proposed by F. P. MATZ, Sc. D., Ph. D., Reading, Pa.

If P be a point within the scalene triangle, such that $\angle PAB=\angle PBC=\angle PCA=\phi$, then $\cot\phi=\cot A+\cot B+\cot C$.........(1), and $\operatorname{cosec}^2\phi=\operatorname{cosec}^2A+\operatorname{cosec}^2B+\operatorname{cosec}^2C$.........(2).

I. Proposed by W. J. GREENSTREET, M. A., Stroud, England.

Let $\angle PAB=\angle PCA=\angle PBC=\phi$. Then $\angle APB=\pi-(\phi+B-\phi)=\pi-B$. Draw PD, PE, PF perpendicular to BC, CA, AB, respectively.



$$PD=PB\sin\phi=\frac{AB\sin PAB}{\sin APB}.\sin\phi=\frac{c\sin^2\phi}{\sin B}$$

$$=\frac{2\,Rc}{b}\sin^2\phi.\quad \text{So } PE=2R\frac{a}{b}\sin^2\phi,$$

$$PF=2R\frac{b}{a}\sin^2\phi.$$

$$\frac{\sin(A-\phi)}{\sin\phi}=\frac{PE}{PF}=\frac{a^2}{bc}=\frac{\sin A\,\sin(B+C)}{\sin B\,\sin C}.$$

$\therefore \cot\phi-\cot A=\cot B+\cot C$, or $\cot\phi=\Sigma\cot A$.

Also $\cot^2\phi=\Sigma\cot^2A+2\Sigma\cot B\cot C$, $\operatorname{cosec}^2\phi-1=\Sigma\operatorname{cosec}^2A-3+2$; *i. e.*, $\operatorname{cosec}^2\phi=\Sigma\operatorname{cosec}^2A$.

II. Solution by the PROPOSER.

Let $PA=m$, $PB=n$, $PC=p$. $\sin(\pi-B):\sin(B-\phi)=c:m$.

$\therefore \cot\phi-\cot B=(m/c\sin\phi)=2\cot B$ ....(1).

Also, $\cot\phi-\cot C=(p/a\sin\phi)=2\cot C$..........(2);

and $\cot\phi-\cot A=(n/b\sin\phi)=2\cot A$..........(3).

Adding, and dividing by (3), we have $\cot\phi=\cot A+\cot B+\cot C$..........($A$).

Squaring (A), and transforming into cosecants, we have

$$\operatorname{cosec}^2\phi=\operatorname{cosec}^2A+\operatorname{cosec}^2B+\operatorname{cosec}^2C.$$

Also solved by M. E. Graber, J. Scheffer, and A. H. Holmes.